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Традиционная международная апрельская математическая конференция в честь Дня работников науки Республики Казахстан, посвященная 75-летию академика НАН РК Тынысбека Шариповича Кальменова.

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4. So, due to proven convexity of squares and from norm properties, we obtain:

$$I(v) = \|\alpha Bu(v_1) + (1-\alpha)Bu(v_2) - z\|^2 \leq \|\alpha Bu(v_1) + (1-\alpha)Bu(v_2)\|^2 \leq \tag{5}$$

$$\leq (\|\alpha Bu(v_1)\| + \|(1-\alpha)Bu(v_2)\|)^2$$

5. From (5), going to scalar product form and estimating, we finally obtain:

$$I(v) = \|\alpha Bu(v_1) + (1-\alpha)Bu(v_2) - z\|^2 \leq \|\alpha Bu(v_1) + (1-\alpha)Bu(v_2)\|^2 = \tag{6}$$

$$= (\alpha Bu(v_1) + (1-\alpha)Bu(v_2), \alpha Bu(v_1) + (1-\alpha)Bu(v_2)) < \alpha I(v_1) + (1-\alpha)I(v_2) \leq$$

$\leq \frac{1}{2}I(v_1) + \frac{1}{2}I(v_2) = \min I$.

So, assuming two minima v_1,v_2 existence, we got *contradiction* (for somewhat v, value $I(v)$ is less than global minimum of I, which is impossible). The contradiction proves the theorem, Q.E.D.

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Keywords: solution existence and uniqueness, inverse problems, mathematical geophysics, oil&gas mining, gravimetric monitoring, conditions on the part of boundary.

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THE STEADY STATES AND TRAVELING WAVE SOLUTIONS OF THE MODIFIED HEISENBERG EQUATION

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The steady states and traveling wave solutions of the Heisenberg and M-I spin systems were recently studied in [1], where the steady states and traveling wave solutions of the 1+1 Heisenberg spin system were obtained using spherical coordinates on the unit sphere. Analogous solutions for the M-I 2+1 spin system were also discovered in [1]. We then studied the extension of the Heisenberg system to the corresponding 2+1 Landau-Lifshitz-Gilbert (LLG) equation, which includes a dissipation term with a small parameter λ [2,4]

$$\vec{S}_t = \vec{S} \times \vec{S}_{xx} + \lambda(\vec{S}_{xx} - (\vec{S} \cdot \vec{S}_{xx})\vec{S}), \quad \vec{S} = (u, v, w), u^2 + v^2 + w^2 = 1 \tag{1}$$

Equation (1) is the isotropic case of LLG equation, for which it is known that it can be mapped to a damped, non-local nonlinear Schrödinger equation [2]. Proceeding in a manner analogous to [1] we have also obtained steady states and traveling waves for the the LLG equation. As in [1], the graphs of these solutions can constructed by plotting u, v, w variables as functions of x, or $\eta = x - vt$, respectively. Thus, including the damping parameter the graphs of these solutions can be similarly obtained and graphically illustrated.

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